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ANISOTROPIC STIFFNESS (HW 1)

E-GLASS

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Aerospace Materials (14:650:449:90)

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Goal: Determine stresses and strains in global and material coordinates for a unidirectional composite lamina subjected to in-plane stresses at a given orientation θ.

PROBLEM STATEMENT

INPUT DATA - MATERIAL: E-GLASS

Material properties from Table 6: Elastic Properties of UD Polymer Matrix Composites

Axial Modulus E11:	45000 MPa
Transverse Modulus E22:	12000 MPa
In-Plane Shear Modulus G12:	5500 MPa
Major Poisson's Ratio v12: Minor Poisson's Ratio v21:	0.28 0.07466667

Applied Stresses (Global Coordinates)

Stress σxx:	874 MPa
Stress σyy	814 MPa
Stress τxy:	420 MPa
Material Orientation Θ:	72.6 deg

METHODS OF ANALYSIS

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

where $m = \cos\theta$ and $n = \sin\theta$.

$$[T_1] = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & 2mn \\ n^2 & m^2 & 0 & 0 & 0 & -2mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -mn & mn & 0 & 0 & 0 & m^2-n^2 \end{bmatrix}$$

$$\begin{cases} S_{11} = \dfrac{1}{E_1}, & S_{12} = \dfrac{-\nu_{21}}{E_2}, \quad S_{13} = \dfrac{-\nu_{31}}{E_3} \\ S_{21} = \dfrac{-\nu_{12}}{E_1}, & S_{22} = \dfrac{1}{E_2}, \quad S_{23} = \dfrac{-\nu_{32}}{E_3} \\ S_{31} = \dfrac{-\nu_{13}}{E_1}, & S_{32} = \dfrac{-\nu_{23}}{E_2}, \quad S_{33} = \dfrac{1}{E_3} \\ S_{44} = \dfrac{1}{G_{23}}, & S_{55} = \dfrac{1}{G_{13}}, \quad S_{66} = \dfrac{1}{G_{12}} \end{cases}$$

$$[T_2] = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & mn \\ n^2 & m^2 & 0 & 0 & 0 & -mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -2mn & 2mn & 0 & 0 & 0 & m^2-n^2 \end{bmatrix}$$

Using the Matrix definitions shown above with the given inputs, the stresses and strains can be calculated in two different ways shown to the right.

Variant 1:

$[\sigma_1]=[T_1][\sigma_x]$

$[\varepsilon_1]=[S_1][\sigma_1]$

$[\varepsilon_x] =[T_2]^{-1}[\varepsilon_1]$

Variant 2:

$[S_x] =[T_2]^{-1}[S_1][T_1]$

$[\varepsilon_x] =[S_x][\sigma_x]$

$[\varepsilon_1] =[T_2][\varepsilon_x]$

Results of Analysis

Variant 1 Results at 72.6 deg:

Variant 1:			
Calculation of σ1 in material orientation		**Calculation of ε1 in material orientation**	
σ11:	1059.06522	**s11:**	2.22222E-05
σ22:	628.934778	**s22:**	8.33333E-05
τ12:	-362.00407	**s12:**	-6.22222E-06
		s21:	-6.22222E-06
		s66:	0.000181818
		ε11:	0.019621411
		ε22:	0.045821492
		γ12:	-0.065818923

Calculation of εx in non-material orientation							
εxx:	0.06226042						
εyy:	0.00318249						
γxy:	0.03909441						
	0.0894254	0.91057	0.28536		0.089425395	0.91057	-0.2854
T2 =	0.9105746	0.08943	-0.2854	$T2^{-1}$ =	0.910574605	0.08943	0.28536
	-0.57071357	0.57071	-0.8211		0.570713568	-0.5707	-0.8211

Results of Analysis

Variant 2 Results at 72.6 deg:

Variant 2:													
Calculation of Sx in non-material orientation													
Delta:	0.97909333		**transformed $\bar{Q}$ matrix**								**Inverse ($Sx = \bar{Q}^{-1}$)**		
			$\bar{Q}_{11}$	**$\bar{Q}_{12}$**	**$\bar{Q}_{16}$**		12880.0667	5821.96853	1369.85		8.30651E-05	-1.14188E-05	-2.48456E-06
Q11:	45960.889		**$\bar{Q}_{12}$**	**$\bar{Q}_{22}$**	**$\bar{Q}_{26}$**	=	5821.96853	40556.615	8248		-1.14188E-05	3.28837E-05	-3.23924E-05
Q22:	12256.2371		**$\bar{Q}_{16}$**	**$\bar{Q}_{26}$**	**$\bar{Q}_{66}$**		1369.84656	8248.00451	7890.22		-2.48456E-06	-3.23924E-05	0.000161032
Q12:	3431.74638												
Q66:	5500												

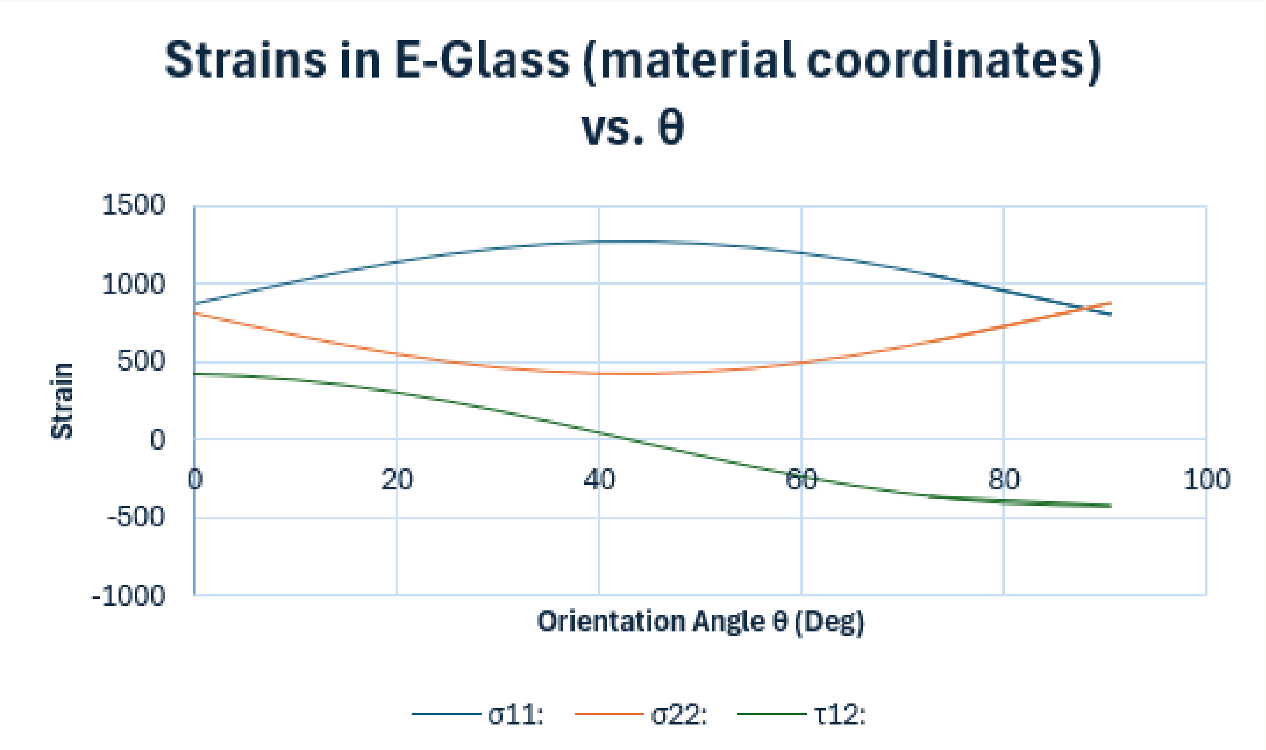
Calculation of εx in non-material orientation	
εxx:	0.06226042
εyy:	0.00318249
γxy:	0.03909441

Calculation of ε1 in material orientation	
ε11:	0.01962141
ε22:	0.04582149
γ12:	-0.0658189

Results of Analysis

Both Variant 1 and Variant 2 yielded the same results at the specified control angle, demonstrating consistency between the two computational approaches.

RESULTS OF ANALYSIS

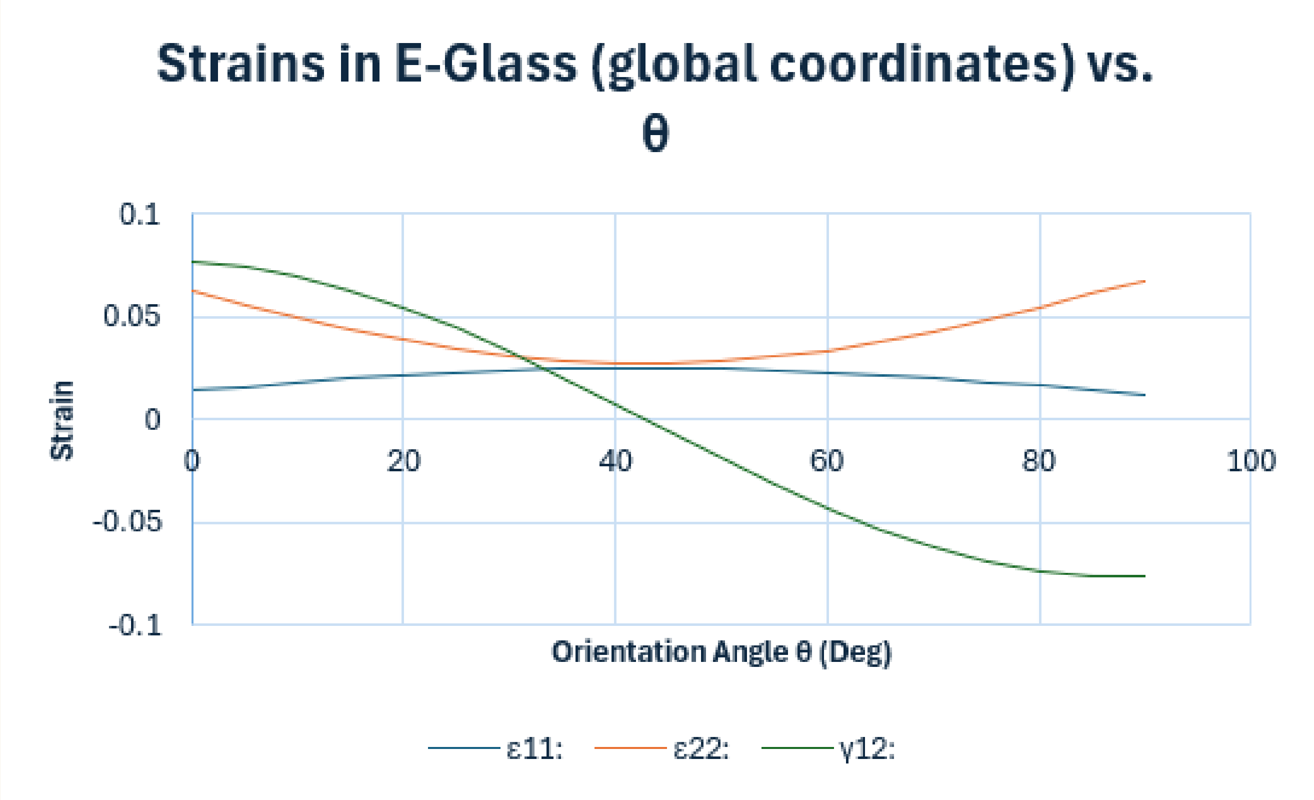


Min σ11:	814	**Max σ11:**	1264
Min σ22:	424	**Max σ22:**	874
Min τ12:	-420	**Max τ12:**	420

- The stresses in the material coordinate system vary significantly with orientation angle θ.
- The longitudinal stress σ_{11} reaches its maximum near 45° and minimum at 0° and 90°.
- The transverse stress σ_{22} decreases with increasing angle, while the shear stress τ_{12} changes sign and reaches its largest magnitude near 90°.
- This behavior reflects the anisotropic stiffness of E-Glass.

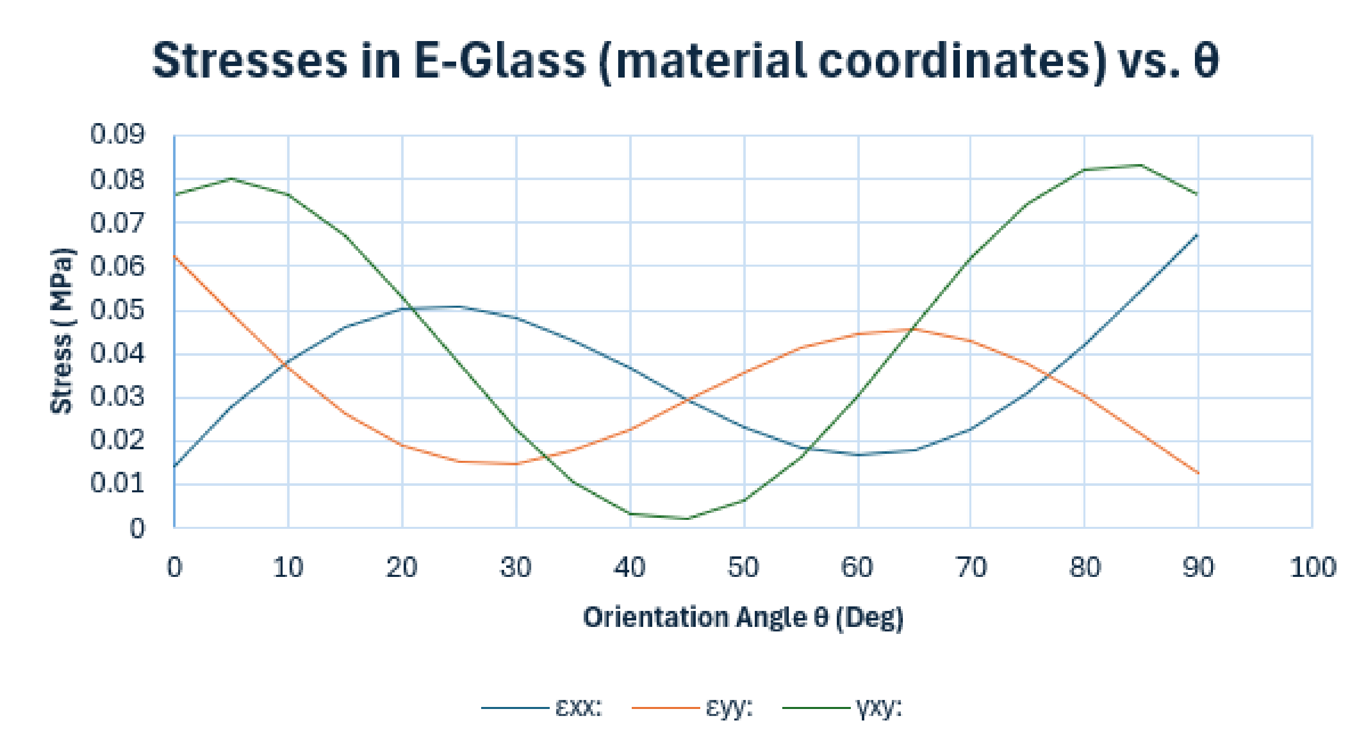
RESULTS OF ANALYSIS

- The global strains εxx, εyy, and γxy vary nonlinearly with orientation angle θ.
- The normal strain εxx increases with angle, while εyy shows a decreasing trend.
- The shear strain γxy changes sign and reaches its largest magnitude at high angles, indicating strong coupling between normal and shear deformation in anisotropic materials.



Min ε11:	0.01265067	**Max ε11:**	0.02545
Min ε22:	0.02746844	**Max ε22:**	0.06777
Min γ12:	-0.07636364	**Max γ12:**	0.07636

RESULTS OF ANALYSIS



Min εxx:	0.01436	**Max εxx:**	0.06777
Min εyy:	0.01265	**Max εyy:**	0.0624
Min γxy:	0.00202	**Max γxy:**	0.08306

- In the material coordinate system, the strain components depend strongly on orientation.
- The longitudinal strain ε11 increases with angle, while the transverse strain ε22 shows a nonlinear variation.
- The shear strain γ12 changes sign and magnitude, indicating rotation of the principal strain directions relative to the global axes.

RESULTS OF ANALYSIS

The stress and strain responses of the E-Glass lamina vary significantly with material orientation due to anisotropic stiffness. Results obtained using Variant 1 and Variant 2 were identical at the control angle ($\theta = 72.6°$), confirming the correctness of both transformation approaches.

The material coordinate stresses and strains exhibit strong directional dependence. The axial (11-direction) response is largest when fibers align closely with the loading direction, while the transverse (22-direction) response dominates when fibers are oriented away from the load. Shear components reach extreme values at intermediate orientations, indicating coupling between normal and shear behavior.

Analysis of the full angular range shows that the optimal orientation depends on the design objective. To minimize normal strain in the fiber direction, orientations near 0° are preferred. To reduce shear effects, orientations closer to 90° are more favorable. These results highlight the importance of selecting fiber orientation to tailor mechanical performance in anisotropic composite materials.

CONCLUSION

The results demonstrate that E-Glass exhibits significant anisotropic behavior, with mechanical response strongly dependent on orientation angle θ.

The agreement between Variant 1 and Variant 2 confirms the validity of the transformation methods used.

Different stress and strain components reach their extrema at different angles, indicating that no single orientation simultaneously minimizes all responses.

Therefore, the optimal material orientation depends on the specific performance requirements of the structure.

EXCEL FILE PROVIDED AS A SEPARATE ATTACHMENT.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	**Material Orientation θ:**	72.6		**σxx MPa:**	874		**Axial Modulus E11 GPa:**	45	**-> Mpa:**	45000		HW1 – Anisotropic Stiffness		Aerospace Materials (14:650:449:90)		
2	**m cosθ:**	0.29904079		**σyy Mpa:**	814		**Transverse Modulus E22 GPa:**	12	**-> Mpa:**	12000		Material: E-Glass		By Avanthika Vuppala		
3	**n sinθ:**	0.95424033		**τxy Mpa:**	420		**In Plane Shear Modulus G12 GPa:**	5.5	**-> Mpa:**	5500						
4							**Major Posson's Ratio v12 :**	0.28								
5							**Minor Poisson's Ratio v21:**	0.07466667								
6	**Variant 1:**															
7	**Calculation of σ1 in material orientation**				**Calculation of ε1 in material orientation**			**Calculation of εx in non-material orientation**								
8	**σ11:**	1059.06522			**s11:**	2.22222E-05		**εxx:**	0.062260416							
9	**σ22:**	628.934778			**s22:**	8.33333E-05		**εyy:**	0.003182487							
10	**τ12:**	-362.004075			**s12:**	-6.22222E-06		**γxy:**	0.039094414							
11					**s21:**	-6.22222E-06										
12					**s66:**	0.000181818			0.089425395	0.91057	0.28536			0.089425395	0.91057	-0.28536
13					**ε11:**	0.019621411		**T2 =**	0.910574605	0.08943	-0.28536		$T2^{-1}$ =	0.910574605	0.08943	0.28536
14					**ε22:**	0.045821492			-0.57071357	0.57071	-0.82115			0.570713568	-0.57071	-0.82115
15					**γ12:**	-0.065818923										
16																
17	**Variant 2:**															
18	**Calculation of Sx in non-material orientation**															
19	**Delta:**	0.97909333		**transformed $\bar{Q}$ matrix**								**Inverse (Sx = $\bar{Q}^{-1}$)**				
20				$\bar{Q}_{11}$	$\bar{Q}_{12}$	$\bar{Q}_{16}$		12880.0667	5821.968533	1369.85		8.30651E-05	-1.14188E-05	-2.48456E-06		
21	**Q11:**	45960.889		$\bar{Q}_{12}$	$\bar{Q}_{22}$	$\bar{Q}_{26}$	=	5821.96853	40556.615	8248		-1.14188E-05	3.28837E-05	-3.23924E-05		
22	**Q22:**	12256.2371		$\bar{Q}_{16}$	$\bar{Q}_{26}$	$\bar{Q}_{66}$		1369.84656	8248.004513	7890.22		-2.48456E-06	-3.23924E-05	0.000161032		
23	**Q12:**	3431.74638														
24	**Q66:**	5500														
25																
26	**Calculation of εx in non-material orientation**						**Calculation of ε1 in material orientation**									
27	**εxx:**	0.06226042					**ε11:**	0.01962141								
28	**εyy:**	0.00318249					**ε22:**	0.04582149								
29	**γxy:**	0.03909441					**γ12:**	-0.0658189								
30	**Strains in Material Coordinates**															

	Column A: θ (degrees)	cosθ:	sinθ:	σ11:	σ22:	τ12:	εxx:	εyy:	γxy:	ε11:	ε22:	γ12:
32	0	1	0	874	814	420	0.014357333	0.06239511	0.076363636	0.01436	0.0624	0.076363636
33	5	0.9961947	0.08716	946.476	741.524	408.4098109	0.027613797	0.04915727	0.079984803	0.01642	0.0559	0.074256329
34	10	0.98480775	0.17365	1015.84	672.161	384.4102964	0.038418691	0.03679642	0.076383217	0.01839	0.04969	0.069892781
35	15	0.96592583	0.25882	1079.98	608.019	348.7306696	0.046088178	0.02650725	0.066776864	0.02022	0.04395	0.063405576
36	20	0.93969262	0.34202	1136.95	551.048	302.4550378	0.050242678	0.01918253	0.053059631	0.02184	0.03885	0.054991825
37	25	0.90630779	0.42262	1185.02	502.978	246.9894628	0.050858023	0.01531601	0.037550534	0.0232	0.03454	0.044907175
38	30	0.8660254	0.5	1222.73	465.269	184.0192379	0.048277516	0.01495469	0.022693822	0.02428	0.03116	0.033458043
39	35	0.81915204	0.57358	1248.93	439.068	115.4576816	0.043182481	0.01770595	0.010746733	0.02502	0.02882	0.020992306
40	40	0.76604444	0.64279	1262.83	425.171	43.38800203	0.036523755	0.0227981	0.0034931	0.02542	0.02757	0.007888728
41	45	0.70710678	0.70711	1264	424	-30	0.029421184	0.02918683	0.002017778	0.02545	0.02747	-0.005454545
42	50	0.64278761	0.76604	1252.41	435.59	-102.4764672	0.023042054	0.03569428	0.006569416	0.02512	0.02851	-0.018632085
43	55	0.57357644	0.81915	1228.41	459.59	-171.8392388	0.018472004	0.04116343	0.016528316	0.02444	0.03066	-0.031243498
44	60	0.5	0.86603	1192.73	495.269	-235.9807621	0.016593029	0.04460901	0.030483327	0.02342	0.03385	-0.042905593
45	65	0.42261826	0.90631	1146.46	541.545	-292.9521294	0.017982562	0.04534618	0.046408432	0.02211	0.038	-0.053264024
46	70	0.34202014	0.93969	1090.99	597.011	-341.0222944	0.022845318	0.04308136	0.061917529	0.02053	0.04296	-0.062004054
47	75	0.25881905	0.96593	1028.02	659.981	-378.7306696	0.030985965	0.03795383	0.074566369	0.01874	0.0486	-0.068860122
48	80	0.17364818	0.98481	959.458	728.542	-404.931505	0.041826042	0.03052301	0.082164799	0.01679	0.05474	-0.07362391
49	85	0.08715574	0.99619	887.388	800.612	-418.8287016	0.054463511	0.02170276	0.083061119	0.01474	0.0612	-0.076150673
50	90	6.1257E-17	1	814	874	-420	0.067768444	0.01265067	0.076363636	0.01265	0.06777	-0.076363636
51	72.6	0.29904079	0.95424	1059.07	628.935	-362.0040749	0.026697492	0.04074624	0.068999898	0.01962	0.04582	-0.065818923