LAMINATED STIFFNESS (HW2)

HM CARBON

BY AVANTHIKA VUPPALA

AEROSPACE MATERIALS

(14:650:449:90)

CONTENT

PROBLEM STATEMENT:

ANALYZE THE STIFFNESS AND DEFORMATION RESPONSE OF A SYMMETRIC LAMINATE COMPOSED OF HIGH MODULUS CARBON PLIES.

LAMINATE STACKING SEQUENCE:
[+Θ / −Θ / +30 / −30]S

Objectives:
- Compute laminate engineering constants
- Determine laminate strains under applied stress
- Determine ply stresses in material coordinates
- Study how laminate behavior varies with orientation angle θ

INPUT DATA - HM CARBON

Table 6: Elastic Properties of Unidirectional Polymer Matrix Composites and 7075-T6 Aluminum

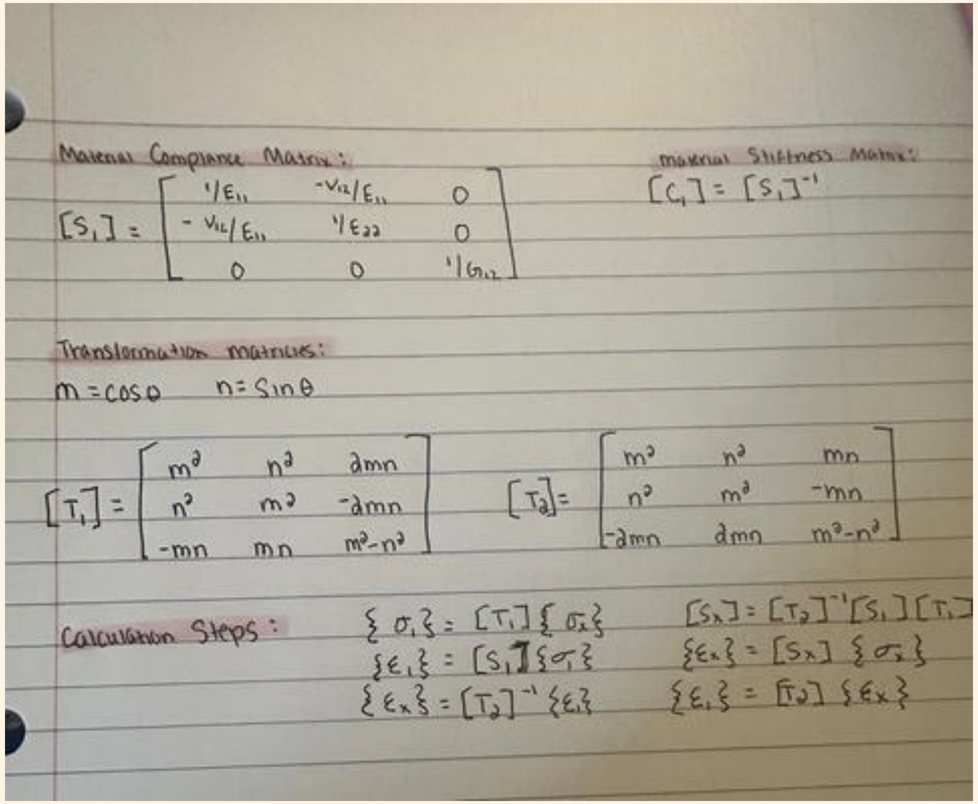
Material Properties:	Value:
E11	225 GPa
E22	7 GPa
G12	3.9 GPa
v12	0.25

Applied Stress State:	Value:
σxx	1799 MPa
σyy	282 MPa
τxy	672 MPa

Ply Thickness: h = 0.0001 m

METHODS OF ANALYSIS

Using the matrix definitions shown and the HM Carbon material properties, laminate strains and ply stresses were computed for different fiber orientation angles. The effective engineering constants were then evaluated as functions of the orientation angle θ.



Material Compliance Matrix:

$$[S_1] = \begin{bmatrix} 1/E_{11} & -\nu_{12}/E_{11} & 0 \\ -\nu_{12}/E_{11} & 1/E_{22} & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix}$$

Material Stiffness Matrix:

$$[C_1] = [S_1]^{-1}$$

Transformation matrices:

$m = \cos\theta$ $n = \sin\theta$

$$[T_1] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \qquad [T_2] = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2-n^2 \end{bmatrix}$$

Calculation Steps:

$\{\sigma_1\} = [T_1]\{\sigma_x\}$

$\{\varepsilon_1\} = [S_1]\{\sigma_1\}$

$\{\varepsilon_x\} = [T_2]^{-1}\{\varepsilon_1\}$

$[S_x] = [T_2]^{-1}[S_1][T_1]$

$\{\varepsilon_x\} = [S_x]\{\sigma_x\}$

$\{\varepsilon_1\} = [T_2]\{\varepsilon_x\}$

Matrix definitions and calculation steps used in the laminate analysis

RESULTS OF ANALYSIS

Ply Stresses:	θ:	(-θ):	(+30):	(-30):
ε11:	0.012403	-0.0009	0.020134	0.008121
ε22:	-0.02264	-0.00933	-0.03037	-0.01836
γ12:	-0.06994	0.077773	-0.05974	0.07361
σ11:	2756.348	-220.335	4485.783	1798.568
σ22:	-137.055	-67.0542	-177.724	-114.531
τ12	-272.764	303.314	-232.979	287.079

Engineering Const:	
Exx:	40396.202
Eyy:	10226.567
Gxy:	48443.36
vxy:	1.5985373
vyx:	0.4046803

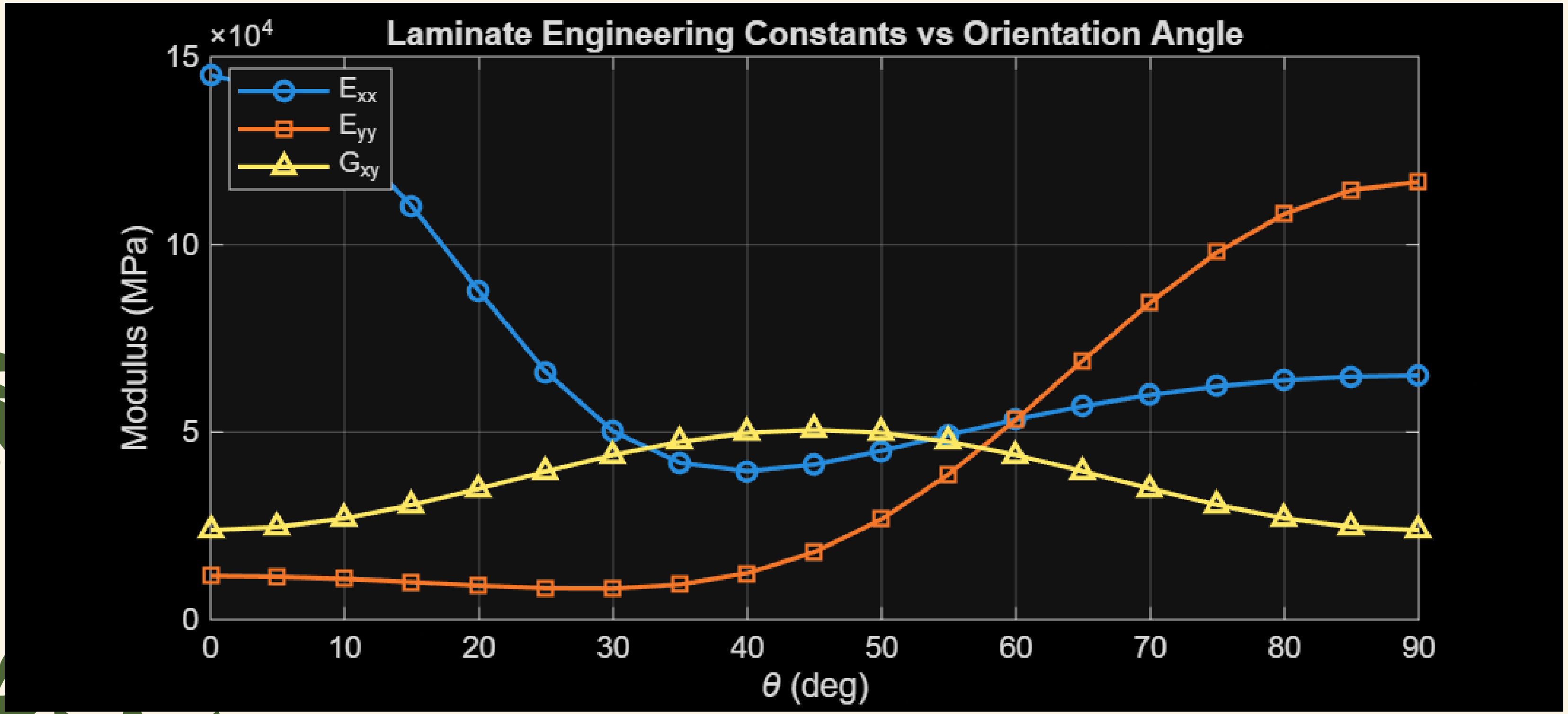
Laminate Strains:	
εxx:	0.033375
εyy:	-0.04361
γxy:	0.013872

RESULTS OF ANALYSIS CONT.

Using the HM Carbon material properties and the laminate stacking sequence $[+\theta/-\theta/+30/-30]s$, the engineering constants, laminate strains, and ply stresses were calculated for the assigned angle $\theta = 36.8°$. The results show that the laminate is much stiffer in the x-direction than in the y-direction, since $E_{xx}$is significantly larger than E_{yy}. The applied stress causes a positive strain in the x-direction and a negative strain in the y-direction, along with a shear strain due to the applied shear stress. The ply stress results also show that each ply carries different stress values depending on its orientation. This highlights how fiber direction plays a major role in how the laminate responds to loading.

RESULTS OF ANALYSIS CONT.

EXX MAX = 145075.21 MPA AT Θ = 0.0 DEG
EXX MIN = 39611.22 MPA AT Θ = 40.0 DEG
EYY MAX = 116661.36 MPA AT Θ = 90.0 DEG
EYY MIN = 8290.89 MPA AT Θ = 30.0 DEG
GXY MAX = 50569.26 MPA AT Θ = 45.0 DEG
GXY MIN = 23901.11 MPA AT Θ = 0.0 DEG



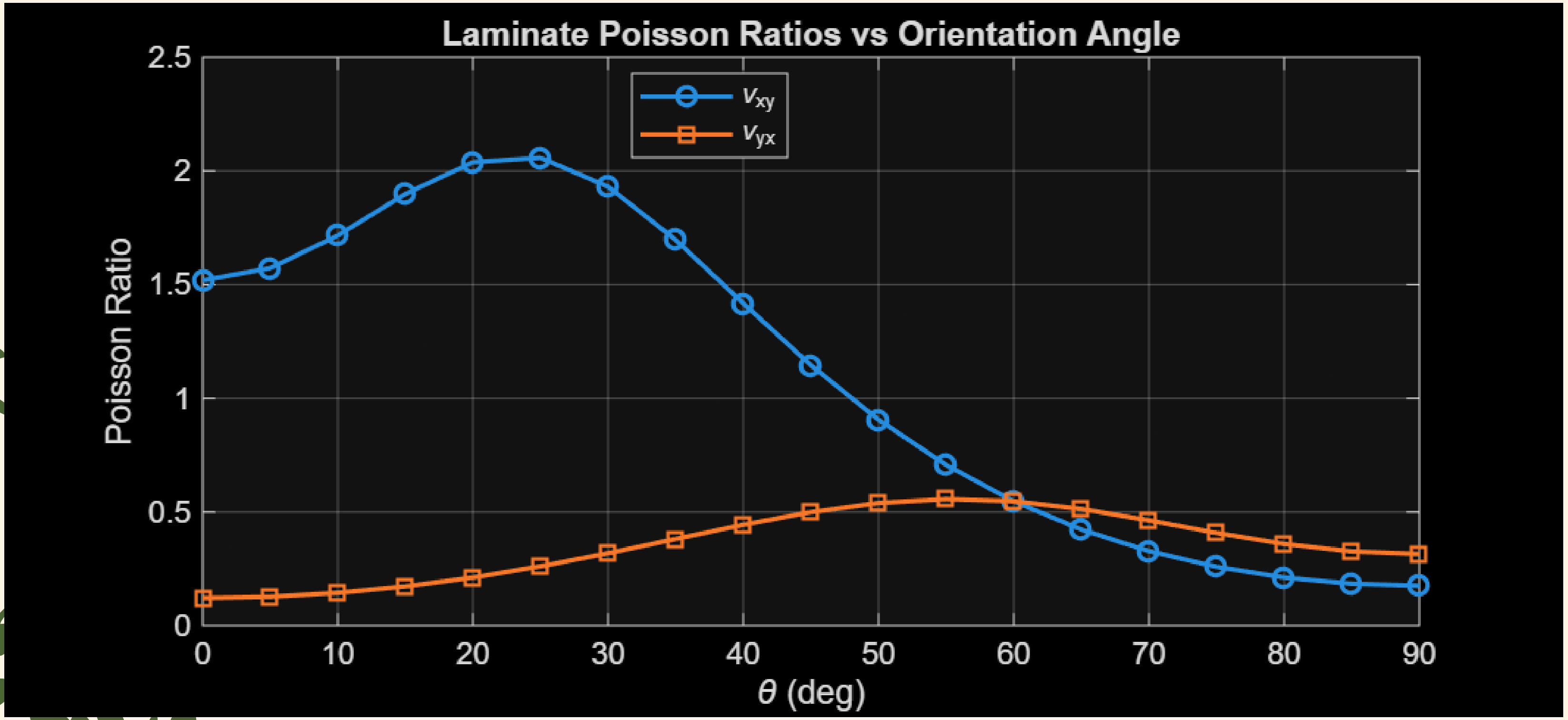
- E_{xx} decreases as θ increases, showing that stiffness in the loading direction drops as fibers rotate away from the x-direction.

- E_{yy} increases toward $90°$,indicating that stiffness shifts toward the transverse direction.

- G_{xy} reaches its maximum near $45°$,where the laminate experiences the greatest shear interaction.

RESULTS OF ANALYSIS CONT.

VXY MAX = 2.0583 AT Θ = 25.0 DEG
VXY MIN = 0.1761 AT Θ = 90.0 DEG
VYX MAX = 0.5572 AT Θ = 55.0 DEG
VYX MIN = 0.1221 AT Θ = 0.0 DEG



- The Poisson ratios vary strongly with fiber orientation, showing the anisotropic behavior of the laminate.

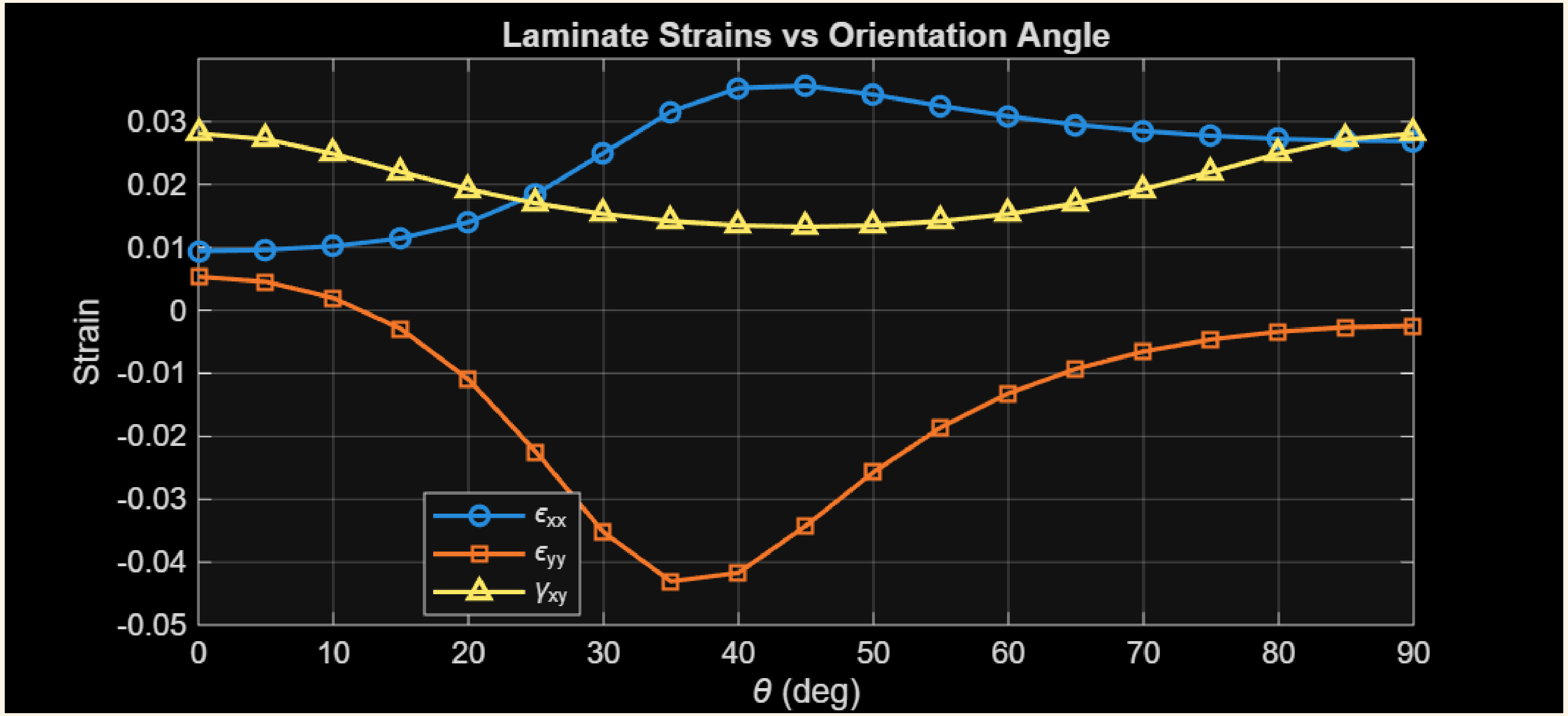
- v_{xy} reaches its maximum near $25°$,while v_{yx} peaks near $55°$,indicating different coupling effects between the x and y directions.

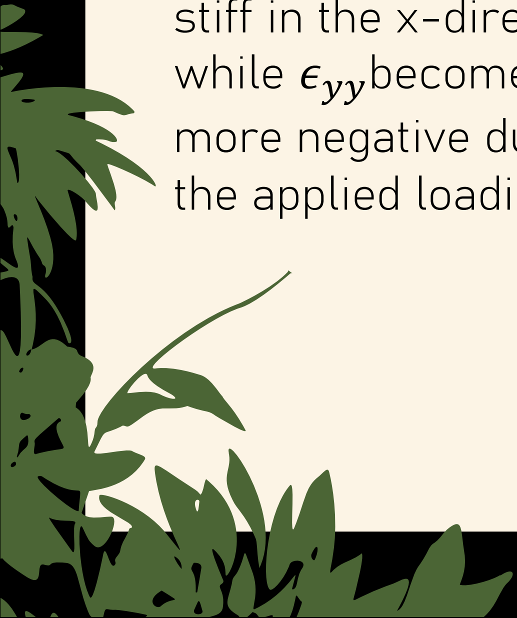
RESULTS OF ANALYSIS CONT.

- The laminate strains change with fiber orientation because the effective stiffness of the laminate varies with θ.

- ϵ_{xx} increases as the laminate becomes less stiff in the x-direction, while ϵ_{yy} becomes more negative due to the applied loading.

EPS_XX MAX = 0.035676 AT Θ = 45.0 DEG
EPS_XX MIN = 0.009447 AT Θ = 0.0 DEG
EPS_YY MAX = 0.005343 AT Θ = 0.0 DEG
EPS_YY MIN = -0.043071 AT Θ = 35.0 DEG
GAMMA_XY MAX = 0.028116 AT Θ = 0.0 DEG
GAMMA_XY MIN = 0.013289 AT Θ = 45.0 DEG



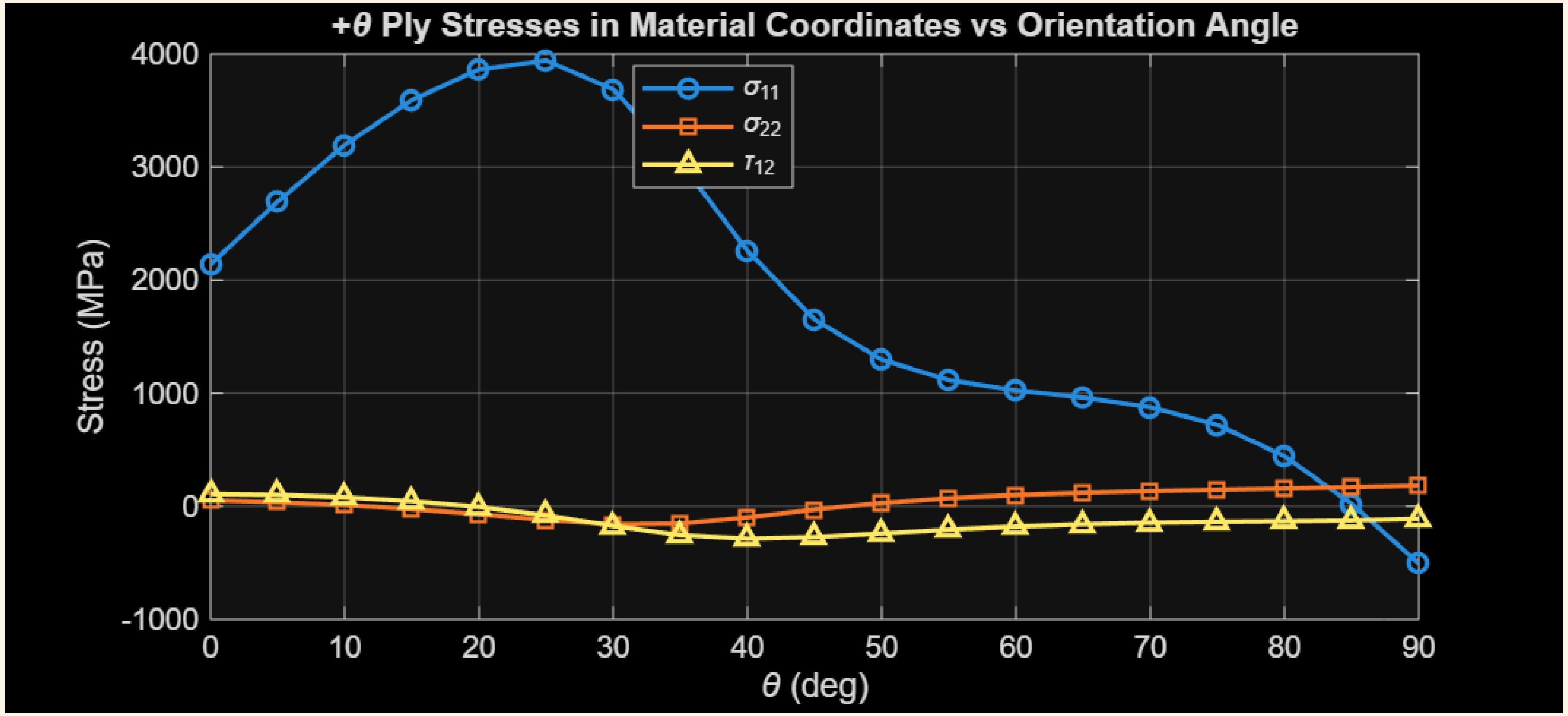


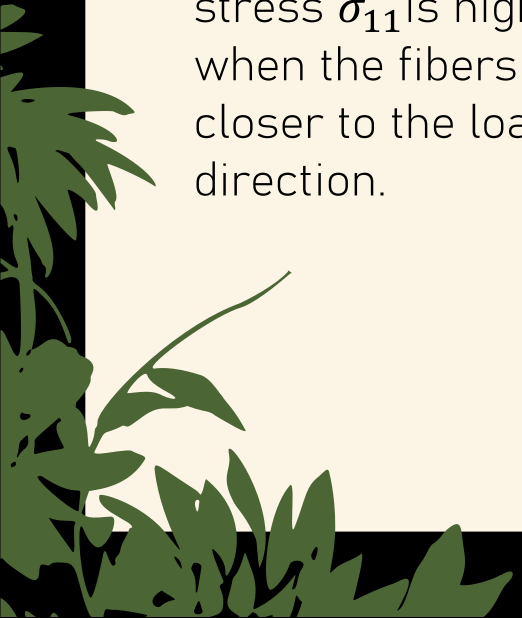
RESULTS OF ANALYSIS CONT.

- Ply stresses vary significantly with fiber orientation because each ply carries load differently depending on its alignment with the applied stress.

- The longitudinal stress σ_{11} is highest when the fibers are closer to the loading direction.

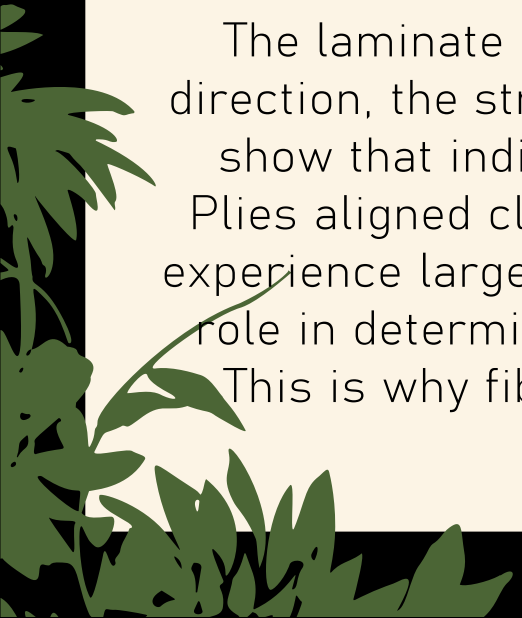
SIGMA11 MAX = 3942.94 MPA AT Θ = 25.0 DEG
SIGMA11 MIN = -505.23 MPA AT Θ = 90.0 DEG
SIGMA22 MAX = 184.20 MPA AT Θ = 90.0 DEG
SIGMA22 MIN = -158.98 MPA AT Θ = 30.0 DEG
TAU12 MAX = 109.65 MPA AT Θ = 0.0 DEG
TAU12 MIN = -286.64 MPA AT Θ = 40.0 DEG





OVERALL DISCUSSION:

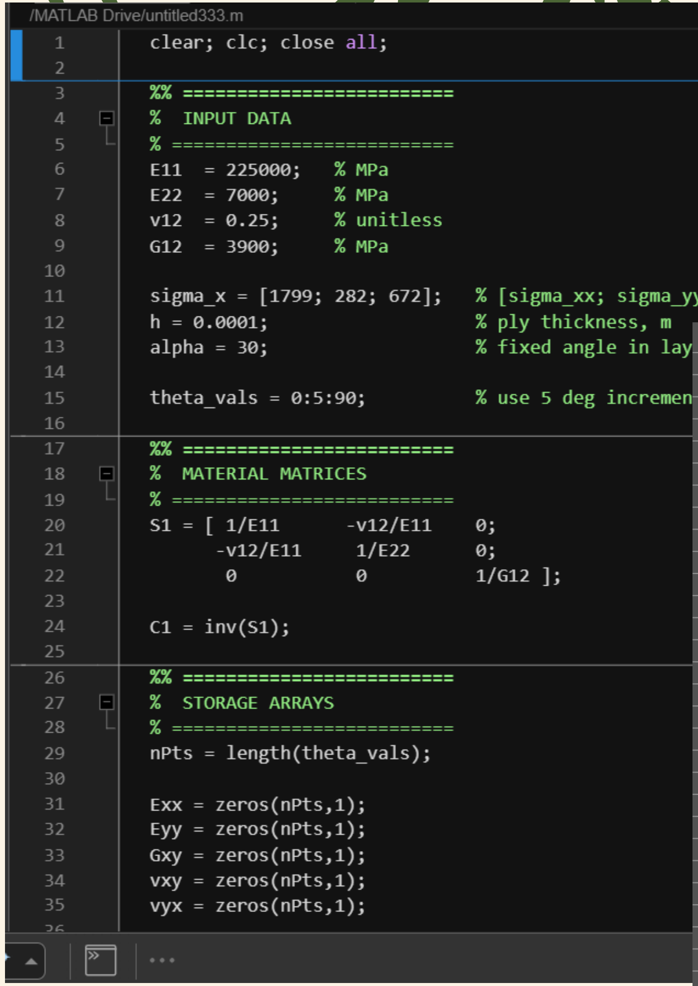
The results show that the mechanical behavior of the laminate depends strongly on the fiber orientation angle θ. As the fiber direction changes, the stiffness properties E_{xx} ,E_{yy} ,and G_{xy}vary significantly. When the fibers are aligned closer to the x-direction, the laminate shows higher stiffness in that direction, which results in lower strain under the applied loading. As the fibers rotate away from the loading direction, the laminate becomes less stiff in the x-direction and the strain increases. The shear modulus reaches its maximum near $45°$,which indicates that shear resistance is strongest when fibers are oriented diagonally relative to the loading direction. The Poisson ratios also change with fiber orientation, showing the anisotropic nature of composite laminates. This means the deformation in one direction is strongly affected by the fiber alignment. The laminate strains follow the same trends as the stiffness values. As stiffness decreases in a direction, the strain in that direction increases under the same applied stress. The ply stress results show that individual plies experience very different stress levels depending on their orientation. Plies aligned closer to the loading direction carry higher longitudinal stresses, while off-axis plies experience larger shear stresses. Overall, these results highlight how fiber orientation plays a major role in determining the stiffness, strain response, and stress distribution of a composite laminate. This is why fiber orientation is a key design parameter when engineering composite structures.



CONCLUSION

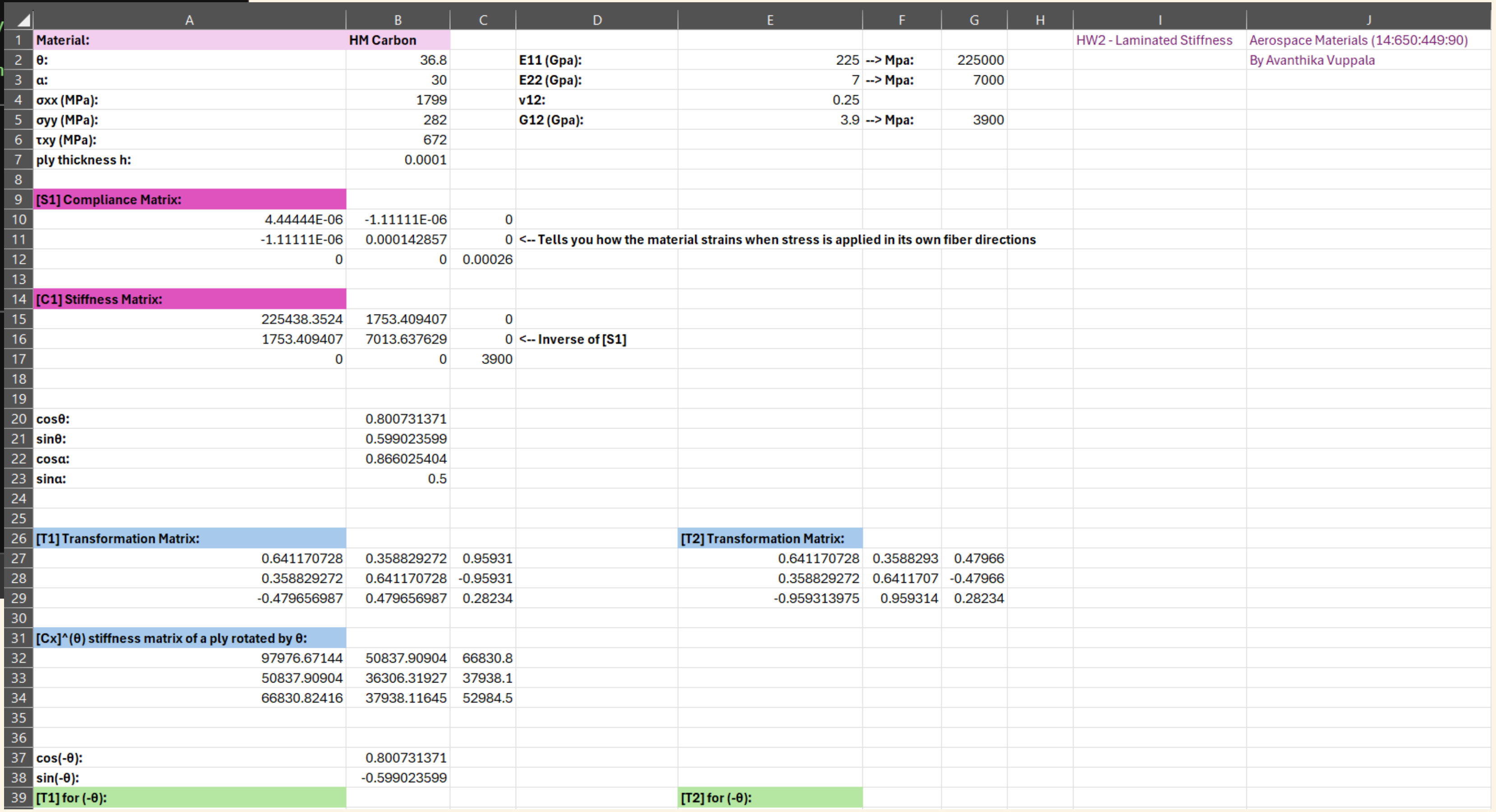
THIS ANALYSIS APPLIED CLASSICAL LAMINATE THEORY TO EVALUATE THE STIFFNESS, STRAIN RESPONSE, AND PLY STRESSES OF A SYMMETRIC HM CARBON LAMINATE. THE RESULTS SHOW THAT LAMINATE MECHANICAL PROPERTIES ARE HIGHLY DEPENDENT ON THE FIBER ORIENTATION ANGLE $\boldsymbol{\theta}$, DEMONSTRATING THE ANISOTROPIC BEHAVIOR OF COMPOSITE MATERIALS. AS THE FIBER ORIENTATION CHANGES, THE EFFECTIVE STIFFNESS SHIFTS BETWEEN THE LONGITUDINAL AND TRANSVERSE DIRECTIONS, WHICH DIRECTLY AFFECTS THE STRAIN RESPONSE OF THE LAMINATE UNDER THE APPLIED LOADING. THE ANALYSIS ALSO SHOWED THAT STRESSES ARE DISTRIBUTED DIFFERENTLY AMONG THE PLIES DEPENDING ON THEIR ORIENTATION, MEANING THAT SOME PLIES CARRY HIGHER LOADS WHILE OTHERS EXPERIENCE GREATER SHEAR EFFECTS. THESE RESULTS HIGHLIGHT HOW FIBER ORIENTATION CAN BE USED AS A DESIGN PARAMETER TO TAILOR THE MECHANICAL PERFORMANCE OF COMPOSITE LAMINATES FOR SPECIFIC LOADING CONDITIONS. OVERALL, THIS STUDY DEMONSTRATES HOW LAMINATE THEORY PROVIDES A USEFUL FRAMEWORK FOR PREDICTING THE BEHAVIOR OF COMPOSITE STRUCTURES AND GUIDING ENGINEERING DESIGN DECISIONS.

EXCEL AND MATLAB FILE PROVIDED AS A SEPARATE ATTACHMENT.



/MATLAB Drive/untitled333.m

```
clear; clc; close all;

%% ========================
%  INPUT DATA
% =========================
E11  = 225000;   % MPa
E22  = 7000;     % MPa
v12  = 0.25;     % unitless
G12  = 3900;     % MPa

sigma_x = [1799; 282; 672];   % [sigma_xx; sigma_yy; tau_xy] in MPa
h = 0.0001;                   % ply thickness, m
alpha = 30;                   % fixed angle in lay

theta_vals = 0:5:90;          % use 5 deg incremen

%% ========================
%  MATERIAL MATRICES
% =========================
S1 = [ 1/E11      -v12/E11    0;
      -v12/E11     1/E22      0;
       0           0          1/G12 ];

C1 = inv(S1);

%% ========================
%  STORAGE ARRAYS
% =========================
nPts = length(theta_vals);

Exx = zeros(nPts,1);
Eyy = zeros(nPts,1);
Gxy = zeros(nPts,1);
vxy = zeros(nPts,1);
vyx = zeros(nPts,1);
```



	A	B	C	D	E	F	G	H	I	J
1	**Material:**	**HM Carbon**							HW2 - Laminated Stiffness	Aerospace Materials (14:650:449:90)
2	**θ:**	36.8		**E11 (Gpa):**	225	**--> Mpa:**	225000			By Avanthika Vuppala
3	**α:**	30		**E22 (Gpa):**	7	**--> Mpa:**	7000			
4	**σxx (MPa):**	1799		**v12:**	0.25					
5	**σyy (MPa):**	282		**G12 (Gpa):**	3.9	**--> Mpa:**	3900			
6	**τxy (MPa):**	672								
7	**ply thickness h:**	0.0001								
8										
9	**[S1] Compliance Matrix:**									
10	4.44444E-06	-1.11111E-06	0							
11	-1.11111E-06	0.000142857	0	**<-- Tells you how the material strains when stress is applied in its own fiber directions**						
12	0	0	0.00026							
13										
14	**[C1] Stiffness Matrix:**									
15	225438.3524	1753.409407	0							
16	1753.409407	7013.637629	0	**<-- Inverse of [S1]**						
17	0	0	3900							
18										
19										
20	**cosθ:**	0.800731371								
21	**sinθ:**	0.599023599								
22	**cosα:**	0.866025404								
23	**sinα:**	0.5								
24										
25										
26	**[T1] Transformation Matrix:**				**[T2] Transformation Matrix:**					
27	0.641170728	0.358829272	0.95931		0.641170728	0.3588293	0.47966			
28	0.358829272	0.641170728	-0.95931		0.358829272	0.6411707	-0.47966			
29	-0.479656987	0.479656987	0.28234		-0.959313975	0.959314	0.28234			
30										
31	**[Cx]^(θ) stiffness matrix of a ply rotated by θ:**									
32	97976.67144	50837.90904	66830.8							
33	50837.90904	36306.31927	37938.1							
34	66830.82416	37938.11645	52984.5							
35										
36										
37	**cos(-θ):**	0.800731371								
38	**sin(-θ):**	-0.599023599								
39	**[T1] for (-θ):**				**[T2] for (-θ):**					